Ast/Phys 5022
Fall 2008
Problem Set #2 (due Sept 23)

1. An object is moving with velocity $\vec{u}$ in frame S. Frame S' is moving with respect to S with $\vec{v}$ in the $+x$-direction.

(a) Write down the three spatial components of vector $\vec{u}'$ (i.e. what is seen by observers in frame S') in terms of $|\vec{v}|$ and the spatial components of $\vec{u}$. Use $u'_1 = dx'/dt'$, $u'_2 = dy'/dt'$, etc, and Lorentz transformations.

(b) In a given inertial frame two particles are shot out simultaneously from a point, with equal speeds w, in orthogonal directions. What is the speed of each particle relative to the other?

2. The metric for a 2D space of the surface of a sphere, expressed in spherical coordinates is

$$dl^2 = r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2,$$

where $r\theta$ and $r\phi$ are the two coordinate distances in that space.

(a) What are the metric elements, i.e. $g_{\theta\theta}$ and $g_{\phi\phi}$?

(b) In general, the proper volume of a 2D space can be calculated from its metric as

$$V = \int g_{11}\, dx_1 \int g_{22}\, dx_2,$$

with appropriate limits on the integrals. What is the total "volume" of the 2D space given above?

(c) Suppose you are sitting at the pole, where $\theta = 0$. What is the circumference of a circle centered on you and having coordinate radius $r\theta$?

(d) Show that for a small enough patch around you the circumference in (c) is just what you would expect in a flat space.