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Part I

Early Universe and the Thermal History

1 Introduction

The early history of the Universe is often called thermal history because the Universe is a dense soup of relativistic particles, that cools down gradually. This early epoch also witnesses a number of phase transitions. At these early times the Universe is very smooth, with density inhomogeneities $\delta\rho/\rho \ll 1$.

The phase transitions came to an end with the quark-hadron transition, often called the QCD transition. Recall that at lower temperatures particles prefer more bound states, so quarks got bound into hadrons. There are two types of hadrons: baryons, which consist of three quarks or anti-quarks (examples are protons and neutrons), and mesons, which are quark-antiquark pairs. This happened at $T \sim$1 GeV = 10^{13} K, and $t = 10^{-5}$sec. The thermal epoch continues; it is characterized by freeze-out of various particle species, as discussed in Section 1.2. The thermal epoch comes to an end when the radiation energy density becomes sub-dominant to the matter energy density, at matter-radiation equality.

1.1 Phase transitions

Very early Universe was very hot, and it has been cooling, though possibly at different rates, ever since then. During the early stages the temperature went through a large range or values (many orders of magnitude). From conventional solid state physics we know that temperature changes can produce phase transitions, and there are commonly known examples of that: ferro-magnets, water drops condensing out of fog, forming in steam, crystal forming, etc. In all these examples the system goes from a high symmetry state to a lower symmetry. For example in a ferromagnet at a high temperature any one location is equivalent to any other, but this is not the case after T falls below some critical temperature. At that time domains form, with different magnetic orientations in each, separated by topological defects, walls in this case. Note that the energy density in any one of the domains is the same as in all others (though that need not be the case in the wall itself). Loss of overall symmetry and formation of defects is also characteristic of phase transitions that took place in the early Universe.

This generic process of symmetry breaking phase transitions probably took place many times in the early history of the Universe as the Universe expanded adiabatically and the temperature steadily tumbled. In fact there is a lot of indirect evidence that these took place: Grand Unification and Electro-Weak are the well known examples. Their relics, particles and forces, are observed. There may have been a host of other phase transitions that may have been cosmologically important. The early Universe, roughly before quarks become confined into hadrons (QCD transition) can be referred to as the epoch of phase transitions.

Just as phase transitions are a generic prediction, so are the resulting defects. Where are they? It is possible that some have a negligible energy density associated with them, and so they had no real impact on the Universe. However, it is also possible that defects had a profound impact: defects could have been the irregularities in the energy distribution that provided seeds for the formation of structure we see today. However, the evidence, especially the observed characteristics of CMB rule out defects as major contributors to structure formation.

1.2 Thermal History

The gist of the thermal history of the early Universe can be summarized as follows.

Early Universe is a soup of particles and anti-particles that undergo frequent interactions with each other. Consider a single particle species (for example, electrons, or neutrinos, or protons...), and let the rate of the main reaction that keeps the species in thermal equilibrium with other species and the radiation field be Γ, in units of # reactions per unit time (i.e. Γ^{-1} is the mean time interval between reactions). If the interaction rate is much faster than the expansion rate of the Universe, $\Gamma \gg H$, many reactions take place in the span of time it takes the ambient temperature to decrease considerably. So the rapid reaction rate keeps the particles in thermodynamic equilibrium with other species and the radiation field. In such a situation the phase-space density of particles/anti-particles is given by the Fermi-Dirac or Bose-Einstein statistics, depending on whether we are dealing with fermions ("+" sign in eq. 1) or bosons ("−" sign)

$$n(E) \propto [e^{E/kT} \pm 1]^{-1}, \tag{1}$$

The constants that we omitted in the above equation involve statistical weights of particles, the Planck constant h, and c. The number density of particles in space is then the integral over the momentum space, $N \propto \int_0^\infty n \cdot 4\pi\ p^2\ dp$. If the particles are somewhat relativistic or non-relativistic, i.e. their rest energy is most of their energy, then ± 1 in eq. 1 can be ignored, and energy approximated by:

$$E = m\gamma = m\,(1 - v^2)^{-1/2} \approx m + \frac{1}{2}mv^2 = m + \frac{p^2}{2m}. \tag{2}$$

Then the space density of particles is,

$$N \propto e^{-m/kT} \int_0^\infty e^{-p^2/2mkT}\ p^2\ dp = \frac{\sqrt{\pi}}{4}(2mkT)^{3/2}\ e^{-m/kT}, \tag{3}$$

where we have used the definite integral $\int_0^\infty x^2\ e^{-x^2}\ dx = \sqrt{\pi}/4$. Eq. 3 is the Maxwell-Boltzmann distribution. It turns out that this simplified expression will be sufficient for the discussion of the early Universe. The factor of particular importance is the exponential factor, because of its (exponential!) sensitivity to the temperature of the Universe.

If the universe expands very very slowly, the temperature decreases slowly as well, and the reaction rate (which is some function of T depending on reaction type) decreases as well, but the Universe has time to continuously reestablish thermal equilibrium because, by design, the amount of time spent at any given T is long. Eventually, T drops to such a low value that for all practical purposes the density of particles becomes nil, as indicated by the $e^{-m/kT}$ factor. Such a slowly expanding Universe will eventually end up with only photons that will continue to cool and redshift. Since the actual Universe is populated with particles (in addition to photons) our rate of expansion must have exceeded the rate of reactions on a number of occasions in the distant past.

For most of the time thermal equilibrium for all species is a good approximation (even though it is never exactly correct because Universe does not ever stop expanding). However, there were brief periods of non equilibrium and these made our Universe interesting. Expansion rate at early times was often comparable to the reaction rates determining interconversion of particles and decay rates. If the rate Γ of thermalizing reaction for a certain particle species drops below the expansion rate H then the reaction has pretty much ceased to take place, and the comoving space density of the corresponding particles cannot change from now on, hence it is said to be frozen-in. If this freezing-in happens before $e^{-m/kT}$ drives a particle species out of existence, then a certain non-negligible space density of these particles is left behind; the species is said to have "frozen out", or decoupled. Barring any other interactions the density of the species per comoving volume stays the same from then on. Depending on whether the freezing out takes place before or after the species becomes completely non-relativistic, approximately determines its final number density relative to photons. The species is known as a "relic"; there are two types:

• **Hot relics:** If the species is relativistic, then the space number density of the relics is high, comparable to the number density of photons. In that case the details of the freeze-out are unimportant, because for $m \lesssim kT$ the value of the exponential factor is not a very sensitive function of m and T. Neutrinos are a good example of hot relics (see Section 2.5).

• **Cold relics:** If the species is non-relativistic when it decouples then its number density is just a trace—tiny compared to the number density of photons—and details of the freeze-out are important because of the sensitivity to exact value of the exponential Boltzmann factor, $e^{-m/kT}$. These relics are called cold relics. Neutron, protons, and light chemical elements (Section 2.6) are good examples.

2 Main events of the Early Universe

2.1 From Planck era to GUT transition

▷ Begin: $t \simeq 10^{-43}$s, $T \simeq 10^{19}$GeV$\simeq 10^{32}$K
▷ End: $t \simeq 10^{-34}$s, $T \simeq 10^{14}$GeV$\simeq 10^{27}$K

All forces but gravity are unified, i.e. strong and electroweak. The force is mediated by superheavy bosons, $m \sim 10^{15}$ GeV.

At the GUT transition: Electro-Weak and Strong forces emerge, and quarks (which interact mostly through the strong force) and leptons (which interact mostly through the weak force), and their anti-particles, acquire individual identities. A generic prediction of GUT is that protons should decay; the time scale varies between theories, but is of the order of 10^{30} seconds. Experimental lower limit on the life-time of proton is around 10^{32} seconds.

The topological defects produced in this transition are magnetic monopoles, in some GUT theories. Approximately, there will be one of these per $1/H_{GUT}$ region, so today monopoles are expected to be very numerous. Annihilation of north and south monopoles is not significant in the most popular versions of GUT (it is inefficient because it takes place via magnetic forces, which are very weak).

Inflation was originally introduced to get rid of the magnetic monopole problem. So inflation, if it happens at all, should take place right after this epoch. Baryogenesis could follow soon after inflation; in fact, that's the earliest time it could have taken place.

2.2 Baryogenesis

The present day Universe is almost certainly not symmetric in baryons, i.e. there are many protons and neutrons, but very few, if any, "naturally occurring" anti-protons, and anti-neutrons. Why is this the case? There are two possibilities: either this asymmetric situation was an initial condition, beyond our ability to understand, or, the Universe was symmetric very early on, and the asymmetry developed somehow. We'll go with the second option. This means that we now have to come up with a plausible mechanism to create the asymmetry out of an initially symmetric situation.

In keeping with that we assume that right after inflation the number of each type of particle produced was exactly balanced by an equal number of corresponding anti-particles. Say, one species produced were X and $\bar{X}$ bosons. Suppose these can decay into quarks, q, and leptons, l, and their antiparticles, $\bar{q}$, $\bar{l}$. To produce an excess number of baryons (or quarks) over anti-baryons (or anti-quarks), one needs interactions that violate baryon number $B\#$ conservation. Such interactions are not known experimentally, at the present time. However, GUT theories (and there are many variants of GUT) predict such interactions.

So, suppose X and $\bar{X}$ decay violates $B\#$ conservation:

$$X \to q + l \quad (\text{net B\#} = 1/3)$$
$$\bar{X} \to \bar{q} + \bar{l} \quad (\text{net B\#} = -1/3)$$

Leptons and bosons have $B\# = 0$. Quarks carry 1/3 of a baryon number (because there are 3 quarks in each baryon), and anti-particles have negative numbers assigned to them. Lepton number conservation is also violated here, but we will concentrate on baryons. Since X and $\bar{X}$ bosons exist in equal numbers, and since the total rate of decay of X must be equal to the total rate of decay of $\bar{X}$ (because they have equal masses), the above two reactions taken together will not generate a net $B\#$. It appears that $B\#$-non-conserving reactions by themselves are not enough. We need a more complex decay mechanism for our bosons.

Consider the following situation where X and $\bar{X}$ decay via *two* channels each:

$$\begin{array}{llcr} [1]: & X \to q + q & r & 2/3 \\ [2]: & X \to \bar{q} + \bar{l} & 1 - r & -1/3 \\ [3]: & \bar{X} \to \bar{q} + \bar{q} & r' & -2/3 \\ [4]: & \bar{X} \to q + l & 1 - r' & 1/3 \end{array} \tag{4}$$

The last two columns give the rates and the net $B\#$ for each of the reactions. The net number of baryons produced when all four of these proceed at the same time is:

$$\text{net } B\# = (2/3)\, r - (1/3)(1 - r) - (2/3)\, r' + (1/3)(1 - r') = r - r' \tag{5}$$

Note that the *total* rates of decay of X and $\bar{X}$ are equal (= 1), as they should be. The important feature of this set of interactions is that if decay rates r and r' are different, then there will be a net number of baryons produced, because in eq. 4 [1] and [4] will be preferred over [2] and [3]. Interactions [1] & [3], and [2] & [4] are C conjugates of each other, for example, [3] can be obtained from [1] by applying charge conjugation, i.e. replacing all the quantum numbers by their negatives, thereby changing particles to anti-particles. If CP symmetry were obeyed by these reactions, then T (time reversal) symmetry would be obeyed as well because CPT is always conserved. Since the rates of [1] and [3] are different, T is broken ans so is CP. The difference in rates can be very small; it will suffice if the two rates differ by 1 part in 10^8.[1]

One more condition has to be satisfied for the baryon asymmetry to stay around forever: the reactions eq. 4 must proceed in an out-of-equilibrium situation, so that the reverse reactions cannot undo what was achieved with eq. 4. Strict equilibrium implies that number densities of each particle species remain the same at all times. So an equilibrium situation will not be able to produce a *net* excess or deficit of anything. In the real Universe the condition of non-equilibrium already exists, and is due to the Hubble expansion.

[1]CP violation has been observed in one type of decay: that of neutral Kaon (a meson), whose equivalent r and r' rates differ by 1 part in 10^3. The experiments were carried out in the 1960's. Since then several instances of CP violation have been detected using B-mesons.

What we have described above are the three conditions that must be met to produce an excess of quarks over antiquarks. Since there is no definite theory of baryogenesis, the above reactions are only examples, rather than actual reactions that took place. It is not entirely clear when exactly these events took place; the earliest possible time is at the end of GUT, the latest time is probably the Electro-Weak transition. Regardless of the epoch and the exact details of baryogenesis, the 3 conditions presented here must be fulfilled. These were first written down by Andrei Sakharov in the mid to late 1960's:

- reactions must violate B# conservation;
- reactions must break the C and CP symmetry;
- reactions must take place in an out-of-equilibrium setting.

2.3 Electro-Weak transition

$\triangleright\ t \simeq 10^{-11}$s, $T \simeq 0.1 - 1$TeV$\simeq 10^{15}$K

This is when electromagnetic and weak forces part ways. As a result, leptons (e.g., electrons) acquire mass. The corresponding bosons also appear: intermediate vector bosons of electroweak force decay into massive W^+, W^-, and Z_0 bosons mediating weak force, and massless photon, mediating electromagnetic force. The massive $W^\pm$, Z_0 bosons ($m \sim 80 - 90$GeV) decay soon thereafter, at the temperature corresponding to their mass. Photons, being massless, do not.

It is quite possible that baryogenesis took place at this time. Electroweak transition was a first order phase transition, and so proceeded through bubble nucleation: bubbles of new phase formed and grew in a sea of the old phase, until they overlapped; eventually all of space would have been engulfed in the new phase, and the phase transition would have been complete. Baryogenesis could take place in the walls of the bubbles.

2.4 Quark-Hadron transition

$\triangleright\ t \simeq 10^{-5}$s, $T \simeq 1$GeV$\simeq 10^{13}$K

Also called the QCD transition. Quarks can no longer exist on their own; they combine into hadrons (baryons and mesons), glued together by gluons (strong force bosons). Quark confinement commences. Some types of WIMPS[2] could have been made at this epoch.

2.5 Lepton era

$\triangleright$ Begin: $t \simeq 10^{-5}$s, $T \simeq 1$GeV$\simeq 10^{13}$K
$\triangleright$ End: $t \simeq 5$s, $T \simeq 0.5$MeV$\simeq 5 \times 10^9$K

This is sometimes called the lepton era because the main players, neutrinos and electrons, are leptons. The following two events immediately precede light element nucleosynthesis.

Neutrino decoupling:
$\triangleright\ t \simeq 1$s, $T \simeq 1$MeV$\simeq 10^{10}$K.
Electroweak transition which already took place, results in a large density of electrons, neutrinos and an equal number of their antiparticles. Neutrinos are kept in equilibrium with the electrons and the radiation field through $\nu + \bar{\nu} \leftrightarrow e^+ + e^-$, and the electrons are tied to photons through $\gamma + \gamma \leftrightarrow e^+ + e^-$. The first reaction is mediated by the weak force, therefore its rate is relatively

[2] Weakly Interacting Massive Particles, a generic name for a yet to be discovered Dark Matter particle.

slow, and so neutrinos decouple, or freeze out early, while still relativistic. Hence neutrinos are hot relics, and the resulting number density of neutrinos is comparable to that of photons. This neutrino background has been cooling ever since it decoupled at $z \sim 10^{10}$, and so its present day temperature is comparable to that of the CMB today. Such low-T neutrino background would be very difficult to detect.

Electron-Positron annihilation:
$\triangleright$ $t \simeq 5$s, $T \simeq 0.5$MeV$\simeq 5 \times 10^9$K
Soon after neutrinos decouple, most of the electrons and all of the positrons annihilate, $e^+ + e^- \rightarrow \gamma + \gamma$, leaving only a small number of electrons that were produced as a result of baryogenesis. The annihilation takes place while electrons are still relativistic, and so the one way reaction above adds a considerable amount of entropy to the Universe, raising its temperature somewhat.

2.6 Big-bang Nucleosynthesis (BBN)

2.6.1 Introduction

Most elements are cooked up in stars, and their observed abundances agree, at least roughly, with this scenario. One notable exception to the stellar nucleosynthesis picture was known very early on, Helium-4, which is about 25% by mass of all nucleons. Alpher, Bethe and Gamow in the 1940's suggested that the element was produced cosmologically, in the early Universe. However, early Universe of their 'trial' models significantly overproduced ^{4}He; their solution was to bathe the early Universe in a radiation background, the presence of which, as we will see shortly, reduces the amount of ^{4}He produced. They predicted this relic radiation to have $T \sim 5$K today, very close to the actual observed value.

BBN took place between at $t \sim 1 - 100$ seconds corresponding to $T \sim 1 - 0.1$MeV, and $10^{10} \gtrsim z \gtrsim 10^9$. Before BBN starts the density of the Universe is dominated by photons, electrons, neutrinos of 3 families (according to the standard theory) and their anti-particles. There are trace amounts of protons and neutrons, their trace status being the result of baryogenesis. Protons and neutrons are non-relativistic by this time.

Let us consider a quantity that will turn out to be important in BBN, it is the baryon to photon ratio, $\eta = n_b/n_\gamma$. The comoving number density of quarks has not changed since baryogenesis, and the comoving number density of baryons has not changed since their confinement into hadrons, all of which happened much before BBN. The comoving number density of photons has not changed much since the end of BBN. So the ratio η parameterizes different possible universe types (that differ in their value of η), but takes out the dynamical evolution, i.e. n_b and n_γ getting diluted with time as the Universe expands. Using present day's value of n_γ, about 412 per cc, and converting n_b to Ω_B with $\Omega_B = \rho_b/\rho_{crit} = \rho_b/(8\pi G/3H_0^2)$, we get

$$\eta = 273 \times 10^{-10}\Omega_B h^2, \quad \text{or} \quad \Omega_B h^2 = 0.004\eta_{10} \qquad (6)$$

Mass of nucleons is about a GeV, while BBN takes places when the ambient radiations temperature is an MeV or less, so we are safely in the non-relativistic regime for the baryons and $n \propto e^{-m/kT}$ can be used as the number density function for protons and neutrons.

2.6.2 Light element production

To a first approximation primordial BBN is the production of ^{4}He; all other elements, if they are produced at all, are produced in minute quantities compared to ^{4}He.

The first step towards making ^{4}He is to make Deuterium, D. Naively, one would expect that protons and neutrons would combine to form Deuterium when the temperature of the Universe drops to just below the binding energy of D, which is 2.2 MeV. But in fact, even if a Deuterium nucleus tries to form at that time, it will be torn apart, or photo-dissociated immediately, because photons outnumber baryons by 10^9 to 1! Therefore Deuterium, and hence all the other elements with higher atomic weights must wait until the Universe gets cooler, when most of the $\gtrsim 2.2$ MeV photons are gone.

The amount of ^{4}He produced will depend on the relative abundance of neutrons and protons. Using the Boltzmann factor and the difference in the rest masses of a neutron and a proton, $\Delta m = 1.29$MeV, we see that the ratio of the number densities of neutrons and protons, $n/p = e^{-\Delta m/kT}$. The reactions that inter-convert protons and neutrons are the weak interactions, involving neutrinos. The n/p tracks its equilibrium value as along as the weak interaction rate is faster than the expansion rate of the Universe, $\Gamma \gg H$. Around the time that neutrinos decouple, the reaction rate inter-converting protons and neutrons drops below the expansion rate. This happens at $t \sim 1$ sec, when kT reaches 0.8 MeV. The n/p ratio at that time is about 1/6. As time progresses, this ratio continues to drop slowly, due to the decay of free neutrons.

When the temperature of the radiation field drops to $T \lesssim 80$keV, most of the high energy photons that can rip apart Deuterium are gone, so D can finally form. By this time n/p ratio has dropped to $\sim 1/7$. Deuterium is the first step towards forming ^{4}He. Virtually all the neutrons are soaked up into ^{4}He. This is partly because ^{4}He is a very tightly bound nucleus, and also because there are no stable nuclei with higher atomic weights that can be made easily out of readily available p and α-particles (there are no stable nuclei with $A = 5$ or 8.) So to a very good approximation out of every 16 nucleons two n's and two p's will go to make ^{4}He, and the mass fraction of ^{4}He produced is $Y = 0.25$.

Thus the abundance of ^{4}He is determined as a competition between the expansion of the Universe and the rate of weak interaction that inter-converts protons and neutrons. Once the ratio of n/p is established virtually all the neutrons will be incorporated into Helium; elements ^{3}He and D are the result of incomplete nuclear burning.

2.6.3 BBN and the baryon-to-photon ratio

How does the final abundance of various elements depend of the value of the η? Increasing the baryon-to-photon ratio means that there are less photons per baryon, and, in effect, the radiation field is not quite as vicious, so that D can form earlier, when the n/p ratio was higher, and so as a result more ^{4}He would be produced.

Decreasing the baryon-to-photon ratio means that the radiation field is even more intense, and so we would expect the nuclear burning would be highly incomplete, i.e. the Universe would not have the chance to produce as much ^4He but will instead leave behind many intermediate products, like ^3He and D.

^{7}Li is a bit more complex compared to the other atomic nuclei since there is more than one way to get ^{7}Li. One process follows the same story as D and ^{3}He, and predicts a drop in ^{7}Li with increasing η. But another method of generating ^{7}Li picks up at higher baryon densities and increases the resulting abundance for higher values of η.

All the light elements mentioned in the section are, at least in principle good indicators of the η ratio. There is no unique theoretical prediction for the value of this ratio (because we don't know how to what degree CP was broken during baryogenesis, and so how many baryons were produced), so at this point we cannot compare its observed value to theoretical value. However, we can check if abundances of each of these elements are self-consistent, i.e. if they agree with the same value of η. And it fact, they do, at least to within $\sim \pm 2\sigma$. So, BBN is an overconstrained theory, and yet it yields consistent results: all 4 elemental abundances agree with the same range of η, and hence $\Omega_B\ h^2$. Furthermore, that value of $\Omega_B\ h^2$ is, within errors, the same value as recovered from the analysis of the CMB: a pleasing confirmation of Big Bang. Current observational value is $\Omega_B\ h^2 \approx 0.022$.

Note that just like the uniformity of the observed CMB, the uniform elemental abundances in various distant regions of the Universe require that the whole of our presently observable Universe had already been in causal contact prior to BBN, which took place much earlier than CMB production.

3 Dark Matter

We saw that baryons amount to about 4% of today's closure density, $\Omega_B = \rho_B/\rho_{crit} \sim 0.04$, where $\rho_{crit} = 3{H_0}^2/8\pi G$. We can observationally estimate the total amount of matter in the Universe, Ω_m. For example, we can measure masses of different types of galaxies and clusters of galaxies, using either dynamical means, or gravitational lensing. Next, we multiply typical masses by the spatial number densities of these objects to get the mass density. Comparing with ρ_{crit} gives Ω_m. There are several variations on this basic measurement theme, but the result obtained is always the same $\Omega_m \approx 0.25 - 0.3$. Comparing this to Ω_B we conclude that most of the mass in the Universe is non-baryonic. It is dark, since we have not detected radiation at any wavelength that is not accounted for by stars and gas, with spectra that correspond well to the lines arising from elements of the Periodic Table. So there are large quantities of dark matter. Futhermore, DM appears not to interact with matter (except through gravity). The need for dark matter also becomes clear when considering the growth of density perturbations in the Universe between the epoch of matter-radiation equality and the epoch of CMB production. Dark matter must consist of elementary particles, which could well have frozen out during the thermal epoch.

3.1 Constraints on particle mass from observations of galaxies

Suppose neutrinos, or some other type of fermions are dark matter particles. Dark matter dominates the mass of galaxies, so these particles must be able to explain observations of galaxies. Being fermions, the phase-space density of these particles, $n \propto [e^{p/kT} + 1]^{-1}$ is limited from above. To get the total number N of particles in some volume of galaxy, one needs to integrate n over momentum space and position space. Roughly, that's going to be $N = n\,(mv)^3\,R^3$, with v and R being the characteristic velocity and radius of a galaxy, and m is the mass of an individual fermion. The velocity should be either rotational velocity of stars/gas clouds at distance R from the center for a spiral galaxy, or velocity dispersion for an elliptical galaxy. The masses of certain types of galaxies are dominated by dark matter; in that case galaxy's mass $M = N\,m = n\,m^4\,v^3\,R^3$. From the dynamical point of view a galaxy in equilibrium must have its KE and PE balanced, so $GM/R \sim v^2$, so $M = v^2\,R/G$. Next, we equate the two expressions for M, and note that an upper limit on n (because particles are fermions) implies a lower limit on m. Putting in all the constants we get,

$$m_\nu \gtrsim 120 \left(\frac{v}{100\mathrm{km.s}^{-1}}\right)^{-1/4} \left(\frac{R}{\mathrm{kpc}}\right)^{-1/2} \mathrm{eV}. \quad (7)$$

For dwarf galaxies, typical velocities at 1 kpc are about 20 km/s, giving a lower limit $m_\nu \gtrsim 150$ eV. For regular galaxies, like the Milky Way, typical velocities at 1 kpc are higher, resulting in a less stringent lower limit.

Notice that the *lower* limit on m we just derived and the *upper* limit from the previous section are already incompatible. Thus, light fermions, including neutrinos cannot be a major contributor to the dark matter in the Universe.

These arguments rule out neutrinos as the dark matter particle. There is yet another argument against neutrinos as DM. Because they are relativistic at early epochs, they do not cluster easily. So they would tend to wipe out density inhomogeneities that might try to form through self-gravity. The amplitude of the observed power spectrum rules out neutrinos as DM.

3.2 Hot thermal relics

This is an argument that is also applied to neutrinos. If neutrinos decouple while still relativistic (at $z_{\text{freeze-out}} \sim 10^{10}$), then their number density today should be comparable to that of CMB photons. That places an upper limit on the particle mass m_ν, since their mass density cannot amount to more than Ω_ν. For neutrinos we get $\Omega_\nu h^2 \approx \sum m_\nu/(100\text{eV})$ where h is the dimensionless Hubble parameter, and the sum is over the number of neutrino species.

If DM particles decouple while relativistic, a similar argument would apply, with $\Omega_{DM}h^2 \approx m_{DM}/(100\text{eV})$, and since $\Omega_m \approx 0.3$, the DM particle mass would be $\sim 30h^2$ ev.

3.3 Cold thermal relics

If DM particles have masses above 1 MeV, and interact through the weak interaction, then their abundance will be reduced by annihilation, and the temprature T at freeze-out is approximately independent of the mass of the particle. This means that the number density of particles is also independent of particles' mass, and so Ω_c in these particles scales as ${m_c}^{-2}$. This argument gives $m_c \sim$ few GeV, if Ω_c is around 0.3. If DM particles are collisionsless, the above argument breaks down. The mass of the DM particle can be considerably above the GeV range.

4 Matter-Radiation Equality

$\triangleright$ $t \simeq 3 \times 10^4$yrs, $T \simeq 3\text{eV} \sim 10^4$K

Thermal epoch of the Universe comes to an end when energy density in radiation becomes smaller than that in matter. When does that happen? Radiation density and matter energy density scale as a^{-4} and a^{-3} respectively. If we know the ratio of their energy densities today we can find a at the time when their energy densities were equal, and hence the redshift and temperature of that epoch, using $T \propto a^{-1} \propto (1+z)$.

$$1 + z_{eq} = \frac{\rho_M}{\rho_R} = \frac{1.88 \cdot 10^{-29} c^2 \Omega_{matter} h^2}{aT_{CMB}^4 + [\nu \text{ contrib.}]} = 2.4 \times 10^4 \Omega_m h^2 \tag{8}$$

In addition to the photons, the denominator contains the energy density in neutrinos as well; the two contributions are about equal. So $T_{eq} = (2.4 \times 10^4 \Omega_m h^2) \times T_{CMB} \approx 10^4 K$. From then onwards the dynamics of the expansion are dictated primarily by non-relativistic pressureless "dust", which is mostly dark matter.